NONTRIVIAL FIXED POINTS IN THREE-DIMENSIONAL ABELIAN HIGGS MODELS WITH FERMIONS

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ABSTRACT. Exact expressions (in the limit $N \to \infty$) for the renormalization group functions of the general class of three-dimensional Abelian Higgs models with fermions are obtained. They are shown to possess several nontrivial infrared and ultraviolet stable fixed points corresponding to models with nonlinear constraints.

1. Over the past few years the renormalization group (RG) has become a fundamental concept and instrument in the theory of elementary particle interactions at short distances [1] and in the modern theory of critical phenomena [2, 3]. Here, one is primarily aiming to find nontrivial ultraviolet (UV) and infrared (IR) stable fixed points (FPs) of the RG flow governing the scaling behaviour of the theory at high and low energies, respectively. Clearly, to achieve this task one has to employ nonperturbative methods, i.e., systematic expansions of corresponding field theory models essentially different from the naive coupling constant(s) perturbation theory.

During the last few years the $1/N$ expansion, N being the number of 'flavor' or 'color' degrees of freedom, has become one of the most popular and powerful nonperturbative methods (for a review, see [4]). Here we shall apply it in the above RG context to the general class of Higgs models with fermions in space-time (Euclidean space) dimensions $D = 3$ (HMF_3), possessing internal $U(n)_{gauge} \times U(N)_{flavor}$ symmetry. In [5, 6] HMF_3 were shown to possess the following interesting properties:

(i) Straightforward applicability of the standard $1/N$ expansion as an expansion around the saddle points of the corresponding effective action [7];

(ii) Phase transition due to a spontaneous breakdown of the internal symmetry below a critical value T_c of the 'temperature' T (cf. Equations (5) and (6) below);

(iii) Dynamical breakdown of discrete P- (P-$'$ and T-) symmetry(-ies), where P- (P- and T-) are space- (space- and time-) reflection(s) in $D = 3$ Euclidean space (Minkowski space-time);

(iv) Nonperturbative particle spectrum-dynamical mass generation, in particular, dynamical generation of (topological) gauge-invariant mass terms [8] for the gauge field(s) and forming of composite fermions in the 'high-temperature' phase; 'confinement' of the gauge fields and some of the elementary scalar and fermion fields in the 'low-temperature' phase.

In the present letter we shall treat explicitly the case of Abelian HMF_3 (i.e., with U(1) gauge symmetry). Unlike the $D = 4$ case, there is no qualitative differences between the RG properties

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of the Abelian and the non-Abelian HMF_3. Namely, in $D = 3$ both U(1) and SU(n) gauge fields are UV asymptotically free due to the positive mass dimension of the gauge coupling constants. Moreover, the quantization of the non-Abelian topological mass term [8] (cf. (4) below) does not affect the RG analysis.

In Section 2, HMF_3 are defined and some of their properties recalled. In Section 3, the exact (in the limit $N \to \infty$) expressions for the RG functions (Gell–Mann–Low beta functions and anomalous dimensions of the elementary and of some composite fields) of HMF_3 are obtained (details of the calculations will be published elsewhere). Here, it is crucial to put $m_\psi = 0$ from the very beginning, m_ψ being the dynamical mass (not the bare one) of the elementary fermions, in order to get new nontrivial IR FPs★. In Section 4, the FP structure is analyzed. All FP theories turn out to be P-invariant (gauge) theories with nonlinear constraints on the fields (except the Gaussian FP). Finally, let us point out that, since $D = 3$ is the physical space dimension, the IR FPs (Equation (18) below) may correspond to a certain (nontrivial) physical *multicritical* behaviour. For the general notions in the theory of critical phenomena, see [2, 3].

2. HMF_3 are defined by the following (Euclidean space) Lagrangian:

$$\mathcal{L}_{\mathrm{HMF}}(\overline{X}) = \mathcal{L}_\varphi + \mathcal{L}_\psi + N\mathcal{L}_A + N\mathcal{L}_C; \tag{1}$$

$$\mathcal{L}_\varphi = |\nabla_\mu \varphi|^2 + M_\varphi^2 \varphi^*\varphi + \frac{\lambda_1 \mu}{2N}(\varphi^*\varphi)^2 + \frac{\lambda_2}{3N^2}(\varphi^*\varphi)^3, \tag{1a}$$

$$\begin{aligned}\mathcal{L}_\psi = {} & i\bar{\psi}\nabla\!\!\!/^{(\epsilon)}\psi + M_\psi \bar{\psi}\psi - \frac{g_1}{N}(\bar{\psi}\psi)(\varphi^*\varphi) - \frac{g_2}{N}(\bar{\psi}\varphi)(\varphi^*\psi) - \\ & - \frac{g_3}{2N}[(\bar{\psi}\varphi)\mathscr{C}(\bar{\psi}\varphi) + (\varphi^*\psi)\mathscr{C}(\varphi^*\psi) + 2(\bar{\psi}\varphi)(\varphi^*\psi)],\end{aligned} \tag{1b}$$

$$\mathcal{L}_A = \frac{1}{4e_\varphi^2\mu} F_{\kappa\lambda}^2(A) - i\frac{\xi_A}{4}\epsilon_{\kappa\lambda\nu}A_\kappa F_{\lambda\nu}(A) + iB\,\partial_\lambda A_\lambda, \tag{1c}$$

$$\mathcal{L}_C = \frac{1}{8e_C^2\mu}\bar{C}i\partial\!\!\!/C + \tfrac{1}{8}\xi_C\bar{C}C + \frac{i}{2N}[(\bar{\psi}\varphi)C - \bar{C}(\varphi^*\psi)], \tag{1d}$$

$$\overline{\mathscr{M}} = \{\overline{X} \mid \overline{X} = (M_\varphi, \lambda_1, \lambda_2, M_\psi, g_1, g_2, g_3, e_\varphi, e_\psi, e_C, \xi_A, \xi_C)\}, \tag{1e}$$

with (slightly different from [5]) notations:

$$F_{\kappa\lambda}(A) = \partial_\kappa A_\lambda - \partial_\lambda A_\kappa, \qquad \nabla_\mu\varphi_a = \partial_\mu\varphi_a + iA_\mu\varphi_a, \qquad \nabla_\mu^{(\epsilon)}\psi_a = \partial_\mu\psi_a + i\epsilon A_\mu\psi_a,$$

$$\epsilon \equiv e_\psi/e_\varphi{}^{\star\star}, \quad \varphi^*\varphi \equiv \varphi_a^*\varphi_a, \quad \bar{\psi}\varphi \equiv \bar{\psi}_a\varphi_a \text{ etc.},$$

★Otherwise, $m_\psi \to \infty$ in the IR scaling limit, thus reducing HMF_3 to the usual $D = 3$ Higgs model (see [9] for the systematic $1/N$ expansion of the latter).

★★Since electric charge is quantized in Nature, we assume $\epsilon \in \mathbb{Z}$.

$\not\partial \equiv \partial_\mu \gamma_\mu$, $\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$ (Euclidean $D = 3$ Dirac matrices), $\mathscr{C}^{-1}\gamma_\mu \mathscr{C} = -\gamma_\mu^T$, $\mathscr{C}^{-1} = \mathscr{C}^T = -\mathscr{C}$ (charge conjugation matrix), $\overline{C}(x) \equiv C(x)\mathscr{C}$. In (1), μ is an arbitrary common mass scale, so that all coupling constants (except the bare masses) are set dimensionless. The auxiliary field $B(x)$ enforce a Landau gauge condition on the (Abelian) gauge field $A_\mu(x)$. (φ_a), (ψ_a), $a = 1, \ldots, N$, belong to the fundamental representation of U(N). $C(x)$ is a two-component real fermionic field (which becomes a Majorana spinor field in the Minkowski version of (1)). Summation over repeated indices is understood and the latter will often be suppressed for brevity.

On the submanifold $\mathscr{M}_{SS}$ of $\overline{\mathscr{M}}$ (1e):

$$\mathscr{M}_{SS} = \{X_{SS} \mid X_{SS} = (T, g, e, \xi_0)\}.$$

$$\begin{gathered} g_1 = g_3 = g, \quad g_2 = 0, \quad \lambda_1 = -4g^2/T, \quad \lambda_2 = 3g^2, \quad M_\varphi = M_\psi = \mu g/T, \\ e_\varphi = e_\psi = e_C = e, \quad \xi_A = \xi_C = \xi_0, \end{gathered} \tag{2a}$$

$$\mathscr{L}_{HMF}(X_{SS}) = \mathscr{L}_{\text{supersymmetric Higgs}}. \tag{2b}$$

For the phase transition and $1/N$ expansion in the supersymmetric Higgs model, see [10].

Under P-reflection (1) transforms as (cf. [8, 5, 6]):

$$\mathscr{L}_{HMF}^{(P)}(\overline{X}) = \mathscr{L}_{HMF}(\overline{X}^{(P)}), \qquad \overline{X}^{(P)} = (\ldots, -M_\psi, -g_1, -g_2, -g_3, \ldots, -\xi_A, -\xi_C). \tag{3}$$

Let us point out one essential difference of (1) from $\mathscr{L}_{HMF}$ considered in [5]. Unlike [5], here we have added explicit (gauge-invariant) mass terms for A_μ and C on the classical level for the following reason. As shown in [5, 6] already in the leading order of the $1/N$ expansion gauge-invariant mass terms are *dynamically* generated ($i\underset{\sim}{A}_\mu \in \mathrm{su}(n)$ in the general non-Abelian HMF_3):

$$\begin{gathered} \frac{(-i)}{4}\xi_0^{\text{dyn}}\epsilon_{\kappa\lambda\nu}A_\kappa F_{\lambda\nu}(A) + \frac{(-i)}{4}\xi_1^{\text{dyn}}\epsilon_{\kappa\lambda\nu}\,\mathrm{tr}\,(\underset{\sim}{A}_\kappa F_{\lambda\nu}(\underset{\sim}{A}) - i\tfrac{2}{3}\underset{\sim}{A}_\kappa \underset{\sim}{A}_\lambda \underset{\sim}{A}_\nu), \\ \xi_0^{\text{dyn}} = \xi_1^{\text{dyn}} = \tfrac{1}{4}\pi. \end{gathered} \tag{4}$$

For topological reasons [8], ξ_1^{dyn} must be quantized ($\xi_1^{\text{dyn}} = l/4\pi$ = fixed, $l \in \mathbb{Z}$) and thus it is not renormalized by higher-order (in $1/N$) corrections, i.e., its RG β-function is identically zero. In the next-to-leading $1/N$ order ξ_0^{dyn} also does not receive infinite renormalizations. However, infinite renormalizations of ξ_0^{dyn} do appear beyond the next-to-leading order because of explicit breaking of P-invariance (cf. (3)), thus necessitating the introduction of the ξ_A-term in (1c).

Below we shall consider explicitly the following restricted version of (1):

$$\begin{aligned} \mathscr{L}(X) &= \mathscr{L}_\varphi + N\mathscr{L}_A + \mathscr{L}_\psi \big|_{g_3 = 0}, \\ &= \{X \mid X = (M_\varphi, \lambda_1, \lambda_2, M_\psi, g_1, g_2, e_\varphi, \epsilon, \xi_A)\}. \end{aligned} \tag{1$'$}$$

Using the results for the RG functions of (1′) (Equations (12) and (14) below) and of the supersymmetric model (2) (Equations (20) below), one can easily deduce the FP structure of the more general model (1). (This will be dealt with in a subsequent paper.)

In order to construct the $1/N$ expansion of (1′), it must be rewritten by means of auxiliary fields (α, σ – bosonic, ρ – fermionic):

$$\mathcal{L}'(X') = \mathcal{L}'_\varphi + \mathcal{L}'_\psi + N\mathcal{L}_A,$$

$$\mathcal{L}'_\varphi = |\nabla_\mu\varphi|^2 + (\alpha_0 + i\alpha(x))(\varphi^*\varphi - N\mu/T - N\sigma) + N\mu(\tfrac{1}{2}\lambda_1 + \lambda_2/T)\sigma^2 + \frac{N}{3}\lambda_2\sigma^3, \quad (5)$$

$$\mathcal{L}'_\psi = i\bar{\psi}\not{\partial}^{(\epsilon)}\psi + \mu\,\Delta\bar{\psi}\psi - g_1\sigma\bar{\psi}\psi - \bar{\psi}\rho\varphi - \varphi^*\bar{\rho}\psi + \frac{N}{g_2}\bar{\rho}\rho;$$

$$\mathcal{M} \cong \mathcal{M}' = \{X' \mid X' = (T, \Delta, \lambda_2, \lambda_1, g_1, g_2, e_\varphi, \epsilon, \xi_A)\},$$

$$M_\varphi^2/\mu^2 + \lambda_1/T + \lambda_2/T^2 = 0, \qquad \Delta = M_\mu/\mu - g_1/T; \quad (5a)$$

where α_0 is an arbitrary, but fixed, nonnegative constant (in the $1/N$ expansion one sets $\alpha_0 = m_\varphi^2$, the physical φ-mass squared). The 'free' propagators (denoted as $\langle \dots \rangle^{(0)}$ in Equation (7)) of the $1/N$ (Feynman diagram) expansion in both the 'high-' and 'low-temperature' phases of (1), (1′)) or (5), respectively) were derived in [5]. Here, we shall need their form only on the (multi)critical surface $\mathcal{N} \subset \mathcal{M}$: $m_\varphi = 0$, $m_\psi = 0$, $\langle\varphi\rangle = 0$ (where $\langle\varphi\rangle$ is the vacuum expectation value of φ). As follows from [5], this is equivalent to setting in (5):

$$T = T_c \ (T_c - \text{'critical temperature'}),\ \Delta = 0;$$

$$\mathcal{N} = \{Y \mid Y = (\lambda_2, \lambda_1, g_1, g_2, e_\varphi, \epsilon, \xi_A)\}. \quad (6)$$

We have explicitly (in momentum space):

$$\langle\varphi_a\varphi_b^*\rangle^{(0)} = \delta_{ab}/p^2, \quad \langle\psi_a\bar{\psi}_b\rangle^{(0)} = \delta_{ab}\not{p}/p^2, \quad \langle\rho\bar{\rho}\rangle^{(0)} = 16N^{-1}(1+w^2)^{-1}[w + \not{p}(p^2)^{-1/2}],$$

$$\langle\alpha\alpha\rangle^{(0)} = 8N^{-1}(p^2)^{1/2}[v^2 + u(p^2)^{1/2}/\mu]\,Q^{-1}(p), \quad \langle\sigma\sigma\rangle^{(0)} = (8N\mu)^{-1}uv^2Q^{-1}(p),$$

$$\langle\alpha\sigma\rangle^{(0)} = i(N\mu)^{-1}uv^2(p^2)^{1/2}Q^{-1}(p), \quad (7)$$

$$\langle A_\kappa A_\lambda\rangle^{(0)} = N^{-1}8(p^2)^{-1/2}[R^2(p) + \xi^2]^{-1}[(\delta_{\kappa\lambda} - p_\kappa p_\lambda/p^2)R(p) + \xi\epsilon_{\kappa\lambda\nu}p_\nu(p^2)^{-1/2}],$$

with the following notations:

$$u = 8[\lambda_1 + (\lambda_2 - g_1^2)2/T_c]^{-1}, \quad v = 8/g_1, \quad w = 16/g_2, \quad \xi = 8\xi_A, \quad h = 8/e_\varphi^2;$$

$$Q_1(p) \equiv v^2 + (1+v^2)u(p^2)^{1/2}/\mu, \quad R(p) \equiv \tfrac{1}{2}(1+\epsilon^2) + h(p^2)^{1/2}/\mu. \quad (8)$$

All vertices can be read off from the nonquadratic part of (5). In the sequal u, v, w, h, ξ (8) will be used on equal footing as $\lambda_1, g_1, g_2, e_\varphi, \xi_A$ respectively, to parametrize points in $\mathcal{N}$.

3. The general form of the RG equations for (5) (on $\mathcal{N}$(6)) reads:

$$\left\{\mu\, \partial/\partial\mu + h\, \partial/\partial h + \sum_{i=1}^{5} \beta_{u_i}\, \partial/\partial u_i + \sum_{\Omega} \zeta_\Omega \int d^3x j_\Omega(x)\, \delta/\delta j_\Omega(x)\right\} W[\{j_\Omega\}]$$

$$= \int d^3x [\theta_\alpha j_\alpha^3(x) + \theta_{\alpha\sigma} j_\alpha(x) j_\sigma(x) + \theta_\rho \bar{j}_\rho(x) j_\rho(x)], \qquad (9)$$

$$(u_1, \ldots, u_5) \equiv (u, v, w, \xi, \lambda_2), \qquad \{\Omega\} \equiv \{\varphi, \psi, A_\mu, \alpha, \sigma, \rho\},$$

where $W[\{j_\Omega\}] = \ln Z[\{j_\Omega\}]$ denotes the (renormalized) generating functional of connected HMF_3 Green's functions with $j_\Omega(x)$ and ζ_Ω being the corresponding external sources and anomalous field dimensions. Note that the right-hand side of (9) is nonvanishing only when sources for the auxiliary fields are present.

The following statements can easily be proved by induction in successive $1/N$ orders:

$$\beta_u = u(1 - 2\zeta_\sigma), \qquad \zeta_{A_\mu} = 0. \qquad (10)$$

Further we have:

$$\beta_{u_i} = N^{-1}\beta_{u_i}^{(1)} + O(N^{-2}), \quad i = 2, \ldots, 5, \qquad \zeta_\Omega = N^{-1}\zeta_\Omega^{(1)} + O(N^{-1}).$$

Thus, in the next-to-leading $1/N$ order, Equation (9) looks as:

$$(\mu\, \partial/\partial\mu + h\, \partial/\partial h + u\, \partial/\partial u) W^{(1)}_{(L_\varphi, L_\psi, L_A, L_\alpha, L_\sigma, L_\rho)} +$$

$$+ \left[\sum_{i=1}^{5} \beta_{u_i}^{(1)}\, \partial/\partial u_i + \sum_{\Omega} L_\Omega \zeta_\Omega^{(1)}\right] W^{(0)}_{(L_\varphi, \ldots, L_\rho)}$$

$$= 6\theta_\alpha^{(1)}\, \delta_{3L_\alpha} + \theta_{\alpha\sigma}^{(1)}\, \delta_{1L_\sigma} + \theta_\rho^{(1)}\, \delta_{2L_\rho}; \qquad (11)$$

$$W_{(L_\varphi, \ldots, L_\rho)} \equiv (\delta^{L_\varphi}/\delta j_\varphi \ldots \delta j_\varphi^*) \ldots (\delta^{L_\rho}/\delta j_\rho \ldots \delta \bar{j}_\rho) W[\{j_\Omega\}]\,|_{j_\Omega = 0}.$$

The superscripts (0), (1) of $W, \beta_{u_i}, \ldots, \theta_\rho$ denote leading and next-to-leading order in $1/N$, respectively. Now, from (11) by means of the explicit graphical rules (7) one can directly compute all $\beta_{u_i}^{(1)}, \zeta_\Omega^{(1)}$.

Here we shall only present the final results. For $\zeta_\Omega^{(1)}$ we obtain:

$$\zeta_\varphi^{(1)} = \frac{2}{3\pi^2}[(1 + v^2 I_u)(1 + v^2)^{-1} + 4(1 + w^2)^{-1} - 16I \cdot (1 + \epsilon^2)^{-1}],$$

$$\zeta_\psi^{(1)} = \frac{2}{3\pi^2}[(1 - I_u)(1 + v^2)^{-1} + 4(1 + w^2)^{-1} - 4I \cdot \epsilon^2 (1 + \epsilon^2)^{-1}],$$

$$\zeta_\alpha^{(1)} = -2\zeta_\varphi^{(1)} - 4(1 + v^2 I_u)[\pi^2(1 + v^2)]^{-1} + 16(1 - w^2)[\pi^2(1 + w^2)^2]^{-1} -$$

$$- 64(1 + \xi\, \partial/\partial\xi)J \cdot [\pi^2(1 + \epsilon^2)^2]^{-1}, \tag{12}$$

$$\zeta_\sigma^{(1)} = -\zeta_\alpha^{(1)} - 32w\,[\pi^2 v(1 + w^2)^2]^{-1} + 32\epsilon^2(\partial/\partial\xi I)[\pi^2 v(1 + \epsilon^2)]^{-1},$$

$$\zeta_\rho^{(1)} = -\zeta_\varphi^{(1)} - \zeta_\psi^{(1)} - 4w^2[\pi^2(1 + w^2)^2]^{-1}\{(1 + v^2)^{-1}[w^2 + 4v^2 + 6vw +$$

$$+ (2(1 - v^2) - 6vw + v^2 w^2) I_u] + 8I \cdot \epsilon(1 + \epsilon^2)^{-1} + 16\xi w J \cdot \epsilon(1 + \epsilon^2)^{-2}\},$$

where the following notations are used:

$$I \equiv I(a, b) = \int_0^\infty dz\, 2z(1 + az)(1 + z^2)^{-2}[(1 + az)^2 + b^2]^{-1},$$

$$J \equiv J(a, b) = \int_0^\infty dz\, 2z(1 + z^2)^{-2}[(1 + az)^2 + b^2]^{-1}, \tag{13}$$

$$I_u \equiv I(u(1 + 1/v^2), 0), \quad a \equiv 2h(1 + \epsilon^2)^{-1}, \ b = 2\xi(1 + \epsilon^2)^{-1}.$$

For the β-functions we get ($\beta_u^{(1)}$ being alreaay known by (10) and (12)):

$$\beta_v^{(1)} = -2v(\xi_\varphi^{(1)} + \zeta_\psi^{(1)}) - 4v/\pi^2 + 32[w(1 - v^2) + v(1 - w^2)][\pi^2(1 + w^2)^2]^{-1} -$$

$$- 4(1 - I_u)v(1 - v^2)[\pi^2(1 + v^2)]^{-1} + 16vI \cdot \epsilon^2[\pi^2(1 + \epsilon^2)]^{-1} -$$

$$- 32(1 - v^2)\epsilon^2[\pi^2(1 + \epsilon^2)]^{-1}(\partial/\partial\xi I) - 64v(1 + \epsilon^4)[\pi^2(1 + \epsilon^2)^2]^{-1}(1 + \xi\,\partial/\partial\xi)J,$$

$$\beta_w^{(1)} = 2w(\zeta_\varphi^{(1)} + \zeta_\psi^{(1)}) - 4w/\pi^2 + 4(1 - I_u)[2v(1 - 3w^2) + 3w(1 - v^2) + v^2 w^3] \times$$

$$\times [\pi^2(1 + v^2)(1 + w^2)]^{-1} + 16wI \cdot (1 + 2\epsilon)[\pi^2(1 + \epsilon^2)(1 + w^2)]^{-1} + \tag{14}$$

$$+ 32\xi(1 - \epsilon - 3\epsilon w^2)J \cdot [\pi^2(1 + \epsilon^2)^2(1 + w^2)]^{-1},$$

$$\beta_\xi^{(1)} = 16\xi[\pi^2(1 + \epsilon^2)^2]^{-1}[(1 + 4\epsilon^4/3)J - \tfrac{2}{3}\widetilde{J}] +$$

$$+ 16v\epsilon^2[3\pi^2(1 + \epsilon^2)(1 + v^2)]^{-1}(I - \widetilde{I}),$$

with the following new notations (a, b same as in (13)):

$$\widetilde{I} = \int_0^\infty dz\, 2z(1+az)(1+z^2)^{-2}[(1+az)^2+b^2]^{-1}v^2[v^2+(1+v^2)uz]^{-1},$$

$$\widetilde{J} = \int_0^\infty dz\, 2z(1+z^2)^{-2}[(1+az)^2+b^2]^{-1}[(1+v^2)u^2z^2+2v^2uz+v^4] \times$$

$$\times\ [v^2+(1+v^2)uz]^{-2}.$$

The rather lengthy full expression for $\beta^{(1)}_{\lambda_2}$ will be omitted here. Only its restrictions on the subspace $\mathscr{K} = \{Y|_{w=0,u,v,\xi=\infty}\} \subset \mathscr{N}$:

$$\pi^2\beta^{(1)}_{\lambda_2}|_{\mathscr{K}} = 16^3 - 32\lambda_2 + \tfrac{3}{4}\lambda_2^2 - 16^{-2}\lambda_2^3 \tag{15}$$

and on the subspace $\mathcal{H} = \{Y|_{u,\xi=\infty}\}$ for small values of $g_1 = 8/v$, $g_2 = 16/w$:

$$\pi^2\beta^{(1)}_{\lambda_2}|_{\mathcal{H}} = -\tfrac{3}{4}[(g_1^2+\tfrac{1}{2}g_2^2)^2+g_1g_2^3] + \tfrac{1}{8}(5g_1^2+2g_2^2)\lambda_2 + \tfrac{3}{4}\lambda_2^2 - 16^{-2}\lambda_2^3 \tag{16}$$

will be needed for the FP analysis. We note also that all $\zeta^{(1)}_{\Omega}$, $\beta^{(1)}_{u_i}$ (except $\beta^{(1)}_{\lambda_2}$) do not depend on λ_2.

4. According to the general theory [2, 3], RG defines an action on $\mathscr{N}$ (6) of the vector field (cf. 9)):

$$X_{\mathrm{RG}} = \sum_{k=0}^{5} \beta_{u_k}\frac{\partial}{\partial u_k}, \quad u_0 = h,\ \beta_{u_0} = h,$$

and, consequently, the IR (UV) FPs of RG are the zeroes Y_* $(Y^*) \in \mathscr{N}$ of β_{u_k} with positive (negative) definite Jacobian ω (eventually, on certain submanifolds):

$$\omega = \|\partial\beta_{u_k}/\partial u_l|_{Y_*^{(*)}}\| > 0\ (<0). \tag{17}$$

Applying criterion (17) to (14), (15) and (16), after simple computations we get the following FP structure of HMF_3 (1′) (recall definitions (8)). First we list the IR FPs:

(a) $(\lambda_2, u, v, w, h, \epsilon, \xi) = (\lambda_2, 0, 0, 0, 0, \epsilon = -1, 0) \equiv Y^{(1)}_* \in \mathscr{N}$ (i.e. λ_2 irrelevant):

$$\mathcal{L}(Y^{(1)}_*) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\,\not\nabla^{(-1)}\psi,\quad \varphi^*\varphi - N\mu/T_C = 0,\ \bar{\psi}\varphi = \varphi^*\psi = 0. \tag{18a}$$

(b) $(\lambda_2, u, g_1, w, h, \epsilon, \xi) = (\lambda_2, 0, 0, 0, 0, \epsilon = -2, 0) \equiv Y^{(2)}_* \in \mathscr{N}$ (i.e., λ_2 irrelevant):

$$\mathcal{L}(Y^{(2)}_*) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\,\not\nabla^{(-2)}\psi,\quad \varphi^*\varphi - N\mu/T_C = 0,\ \bar{\psi}\varphi = \varphi^*\psi = 0; \tag{18b}$$

(c) $(\lambda_2, u, g_1, g_2) = (\lambda_2, 0, 0, 0) \equiv Y_*^{(3)} \in \mathcal{E} = \{Y|_{e_\varphi = 0}\} \subset \mathcal{N}$ (i.e., λ_2 irrelevant, A_μ absent in (5)):

$$\mathcal{L}(Y_*^{(3)}) = |\partial_\mu \varphi|^2 + i\bar{\psi}\partial\!\!\!/\psi, \quad \varphi^*\varphi - N\mu/T_C = 0. \tag{18c}$$

(d) $(\lambda_2, g_1, g_2) = (0, 0, 0) \equiv Y_*^{(4)} \in \mathcal{H} \subset \mathcal{N}$ (Gaussian IR FP):

$$\mathcal{L}(Y_*^{(4)}) = |\partial_\mu \varphi|^2 + i\bar{\psi}\partial\!\!\!/\psi. \tag{18d}$$

The set of the UV FPs is as follows:

(α) $(\lambda_2, g_1, w, e_\varphi) = (\lambda_2, 0, 0, 0) \equiv Y_{(1)}^* \in \mathcal{F} = \{Y|_{u=0}\} \subset \mathcal{N}$ (i.e. λ_2 irrelevant, A_μ decouples from (5)):

$$\mathcal{L}(Y_{(1)}^*) = |\partial_\mu \varphi|^2 + i\bar{\psi}\,\partial\!\!\!/\psi, \quad \varphi^*\varphi - N\mu/T_C = 0, \bar{\psi}\varphi = \varphi^*\psi = 0. \tag{19a}$$

(β) $(\lambda_2, u, g_1, w, e_\varphi) = (\lambda_2^{(*)}, \infty, 0, 0, 0) \equiv Y_{(2)}^* \in \mathcal{N}$:

$$\mathcal{L}(Y_{(2)}^*) = |\partial_\mu \varphi|^2 + (3N^2)^{-1}\lambda_2^{(*)}(\varphi^*\varphi - N\mu/T_C)^3 + i\bar{\psi}\,\partial\!\!\!/\psi, \quad \bar{\psi}\varphi = \varphi^*\psi = 0, \tag{19b}$$

where $\lambda_2^{(*)} \approx 179$ is the unique positive zero of $\beta_{\lambda_2}^{(1)}$ (15).

Finally, a remark concerning the FPs of the supersymmetric Higgs model $\mathcal{L}(X_{SS})$ (2) is in order. Since in $\mathcal{L}(X_{SS})$ explicit (gauge-invariant) mass terms for A_μ, C are present we shall have two β-functions: β_g (or $\beta_v = -8\beta_g/g^2$) and β_ξ (where $v = 8/g$, $\xi = 8\xi_0$, $h = 8/e^2$; g, ξ_0, e as in (2)) instead of only β_v as in [10]. In complete analogy with [10] using superspace★ $1/N$ diagram rules we get ($I = I(h, \xi)$, $J = J(h, \xi)$ as defined in (13)):

$$\beta_v^{(1)} = -\frac{16v}{\pi^2}\left[\tfrac{1}{2}(1+v^2)^{-1} - \tfrac{1}{3}(1-6v^2)(1+v^2)^{-2} - I + (1+\xi\,\partial/\partial\xi)J\right] - \\ - \frac{8}{\pi^2}\xi J + \frac{2}{\pi^2}(1+4v^2)(\partial/\partial\xi\, I), \tag{20a}$$

$$\beta_\xi^{(1)} = -4vI\cdot[\pi^2(1+v^2)]^{-1} - 2\xi J\cdot[6+(1+v^2)^{-1}]\pi^{-2}, \tag{20b}$$

$$\zeta_\Sigma^{(1)} = -\frac{8}{\pi^2}\left[\tfrac{1}{2}(1+v^2)^{-1} + 3v^2(1+v^2)^{-2} - (1+v\,\partial/\partial\xi)I + (1+\xi\,\partial/\partial\xi)J\right]. \tag{20c}$$

Equations (20) should replace Equations (12) of Reference [10] correspondingly. Applying criterion (17) to (20), we obtain the following supersymmetric FPs:

(e) Trivial Gaussian IR FP: $(g, \xi) = (0, \infty) \equiv Y_{SS*}$ (i.e., A_μ, C absent):

$$\mathcal{L}(Y_{SS*}) = |\partial_\mu \varphi|^2 + i\bar{\psi}\,\partial\!\!\!/\psi. \tag{18e}$$

★For a comprehensive review of supersymmetry, see [11].

(γ) Nontrivial UV FP: $(v, e) = (0, 0) \equiv Y^*_{SS}$ (i.e., A_μ, C decouple):

$$\mathcal{L}(Y^*_{SS}) = |\partial_\mu \varphi|^2 + i\bar{\psi}\,\partial\!\!\!/\psi + \frac{T_C}{4N\mu}(\bar{\psi}\psi)^2, \quad \varphi^*\varphi - N\mu/T_C = 0,\ \bar{\psi}\varphi = \varphi^*\psi = 0, \quad (19c)$$

i.e., this FP exactly coincides with the $D = 3$ supersymmetric U(N) nonlinear sigma model [12].

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Erratum and Addendum

'Nontrivial Fixed Points in Three-Dimensional Abelian Higgs Models with Fermions', by E. R. Nissimov and S. J. Pacheva, *LMP* **8** (1984), 239–247. (Received: 2 September 1985).

The results for $\beta_\xi^{(1)}$ in Equations (14) and (20b) are not correct due to the omission of certain graphs in the previous computation. Adding the contributions of the latter, one gets

$$\beta_\xi^{(1)} = 0 \qquad (14'), (20b')$$

in accordance with a recently-proved general theorem [1] on nonrenormalization of the parameter ξ of the gauge-invariant mass term

$$-i\xi/4\,\varepsilon_{\mu\nu\lambda}A_\mu F_{\gamma\lambda}(A)\;(+\tfrac{1}{8}\xi\bar{C}C \text{ in the supersymmetric case})$$

in gauge theories with massive matter fields. In view of (14′), (20b′), the following new fixed points (FP) should be *added* to Equations (18) and (19) (renumbering (19c) as (19g)):

(f) Nontrivial supersymmetric infrared FP: $(g, h) = (0, 0) \equiv Y'_{SS*}$:

$$\mathscr{L}(Y'_{SS*}) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\nabla\!\!\!/\,\psi, \quad \bar{\psi}\varphi - \mathscr{C}\varphi^*\psi = 0\,. \qquad (18f)$$

(γ) Ultraviolet (UV) FP $(\lambda_2, u, v, g_2, \varepsilon, \xi) = (\lambda_2, \infty, 0, 0, |\varepsilon| \leqslant 3, 0) \equiv Y^*_{(3)} \in N_0 = \{Y|_{h=0}\}$ (i.e., λ_2 irrelevant):

$$\mathscr{L}(Y^*_{(3)}) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\nabla\!\!\!/^{(\varepsilon)}\psi + T_c(4N\mu)^{-1}(\bar{\psi}\psi)^2, \qquad \varphi^*\varphi - N\mu/T_c = 0\,. \quad (19c)$$

(δ) UV FP $(\lambda_2, v, w, \varepsilon, \xi) = (\lambda_2, 0, 0, \varepsilon \neq \pm 1, 0) \in \mathscr{F}_0 = \{Y|_{u=h=0}\}$ (i.e., λ_2 irrelevant):

$$\mathscr{L}(Y^*_{(4)}) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\nabla\!\!\!/^{(\varepsilon)}\psi, \qquad \varphi^*\varphi - N\mu/T_c = 0\,, \quad \bar{\psi}\varphi = \varphi^*\psi = 0\,. \quad (19d)$$

(ε) UV FP $(\lambda_2, g_1, w, \varepsilon, \xi) = (\lambda_2, 0, 0, \varepsilon = 1, 0) \in \mathscr{F}_0$ (i.e., λ_2 irrelevant):

$$\mathscr{L}(Y^*_{(5)}) = \mathscr{L}(Y^*_{(4)})|_{\varepsilon=1}\,. \qquad (19e)$$

(ζ) UV FP $(\lambda_2, v, g_2, \varepsilon, \xi) = (\lambda_2, 0, 0, \varepsilon, 0) \in \mathscr{F}_0$ (i.e., λ_2 irrelevant, ε arbitrary):

$$\mathscr{L}(Y^*_{(6)}) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\nabla\!\!\!/^{(\varepsilon)}\psi, \qquad \varphi^*\varphi - N\mu/T_c = 0\,. \qquad (19f)$$

(η) Nontrivial supersymmetric UV FP $(v, \xi) = (0, 0) \equiv Y'^*_{SS}$ ($h \equiv 0$ – fixed):

$$\begin{aligned} &\mathscr{L}(Y'^*_{SS}) = |\nabla_\mu\varphi|^2 + i\bar{\psi}\nabla\!\!\!/\,\psi + T_c(4N_\mu)^{-1}(\bar{\psi}\psi)^2\,, \\ &\varphi^*\varphi - N\mu/T_c = 0\,, \qquad \bar{\psi}\varphi = \varphi^*\psi = 0\,, \end{aligned} \qquad (19h)$$

i.e., Y'^*_{SS} coincides with the $D = 3$ supersymmetric gauged nonlinear sigma model.

The results for the remaining renormalization group functions and FP in Equations (12)–(16) and (18)–(20) are unaltered.

Let us note that although ξ is not renormalized according to (14′) and (20b′), it gives nonzero contributions to the remaining renormalization group functions. As a consequence, if $\xi \neq 0$ the models (18a, b, f) and (19c, d, e, f, h) cease to be FP.

As a final comment, let us point out that, using the $1/N$ expansion together with the 'soft-mass' renormalization scheme described in [2] (and used here), one can extend the proof of [1] about nonrenormalization of ξ in $D = 3$ (Abelian) gauge theories with massive charged fields to the purely massless case. Indeed, the above-mentioned scheme is (a) free of infrared divergences in the massless case (unlike the usual coupling constant perturbation theory) and (b) it yields a smooth zero mass limit of the models with massive matter fields.

This Erratum and Addendum substitutes the previous Erratum (*LMP*, **8** (1984), 349).

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